

# OPERATIONAL USE OF THE HADAMARD VARIANCE IN GPS

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## Abstract

*With upcoming GPS Block IIR launches scheduled, rubidium clock estimation will require more attention than ever before during the next decade of GPS operations. GPS Master Control Station (MCS) estimation architecture relies on a three-state polynomial clock model, which does not include a time-variant decay parameter for frequency drift. Since current GPS rubidium frequency standards exhibit significant time-dependent frequency drift changes, the MCS is compelled to make precise utilization of the random run FM process noise parameter, known as  $q_3$ .*

*The work of various scientists over the past three decades has shown the Hadamard variance to converge for random run FM. At PTTI '95, the 2d Space Operations Squadron (2 SOPS) introduced an algorithm<sup>[1]</sup> that presented a simple, convergent polynomial relationship between the Hadamard variance and the MCS's Kalman filter process noise parameters. Until recently, however, neither the Hadamard variance nor the Hadamard-Q equation had actually been put to use in GPS.*

*The Naval Research Laboratory (NRL) has now created analysis software designed to employ the Hadamard variance in their GPS clock analyses, to supplement their already existing software, which makes use of the Allan variance. This paper presents results of the NRL analysis using both the Allan and Hadamard variances for several operational GPS rubidium frequency standards, as well as results from the recent operational use of the Hadamard-Q equation, by 2 SOPS personnel, based on the NRL analysis data.*

## INTRODUCTION

The three-sample (Hadamard) variance equation has sporadically appeared in publications related to timing over the past three decades<sup>[2]</sup>. In 1995, the 2d Space Operations Squadron (2 SOPS) reviewed the Hadamard variance for its potential use in GPS<sup>[1]</sup>.

$${}_H\sigma^2_y(\tau) = \frac{1}{6(M-2)} \sum_{i=1}^{M-2} (\bar{y}_{i+2} - 2\bar{y}_{i+1} + \bar{y}_i)^2, \quad \bar{y}_i = \text{the time-averaged frequency over } \tau_i. \quad (1)$$

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Since the Hadamard variance converges for two more noise types than the Allan variance, namely  $\alpha = -3$  and  $\alpha = -4$  (Figure 1), the Hadamard variance ideally corresponds to the noise types modeled by the GPS Master Control Station (MCS). 2 SOPS documented this relationship as follows<sup>[1]</sup>:

$$H\sigma_y^2(\tau) = (10/3)q_0\tau^2 + q_1\tau^1 + (1/6)q_2\tau + (11/120)q_3\tau^3 \quad (2)$$

where  $q_1$ ,  $q_2$ , and  $q_3$  are the MCS's continuous time update clock process noise parameters, and  $q_0$  is the parameter designed for representation error. The MCS Kalman filter employs process noise parameters to increase elements in the MCS Kalman filter's state covariance matrix during time update predictions. This compensation effectively models GPS frequency standards as having three individual random walk processes. The representation error parameter helps to compensate for white/flicker PM noise, some, but not all, of which is caused by the frequency standard itself. The relations between the noise types, spectral density exponents, and applicable Kalman filter parameters are tabulated in Figure 1<sup>[1]</sup>.

## NRL HADAMARD VARIANCE SOFTWARE

Since the inception of GPS, the Naval Research Laboratory (NRL) has played an integral role in GPS frequency standard development and analysis<sup>[3]</sup>. In recent years, NRL's on-orbit clock analysis has provided the valuable independent agency support to 2 SOPS necessary to ensure the integrity of GPS, upon which the navigation and timing communities have become dependent.

In the past two years, NRL has become a critical element in the optimization of GPS accuracy. Since October 1994, 2 SOPS has depended on NRL clock stability data as the means for fine-tuning GPS clock estimation in the MCS<sup>[4]</sup>. The unique knowledge and experience levels of NRL personnel contribute to timely, understandable, and very presentable analysis updates, technical notes, and quarterly reports. By interfacing with personnel from 2 SOPS and the United States Naval Observatory (USNO), NRL is able to make intelligent judgments for refining, adjusting, and applying the appropriate corrections to raw data, resulting in analysis documents that are extremely useful and applicable to the GPS frequency standard community.

NRL's analysis capability utilizes data from timing receivers operated by both USNO and the National Imagery and Mapping Agency (NIMA). These data include measurements of GPS frequency standards referenced to the Department of Defense (DoD) Master Clock at USNO. In early 1996, NRL began studying the utility of the Hadamard variance, using both simulated data and real-world GPS frequency standard measurements<sup>[5]</sup>. By modifying already existing software extensively refined for Allan variance analysis, in April 1996, NRL began including Hadamard deviation (the square root of the Hadamard variance) plots in many of their reports to 2 SOPS<sup>[6]</sup>. Since then, NRL has produced Hadamard deviation analyses on all 12 rubidium frequency standards that have been activated on GPS Block II/IIA satellites<sup>[7]</sup>.

Many clock scientists have studied rubidium frequency standard stability using the Allan variance. These studies have usually demonstrated the dominating effect of frequency drift. For GPS Block II/IIA frequency standards, this effect is typically dominant even at  $\tau = 1$  day. Figure 2 presents a composite of Allan deviation plots for several GPS rubidium frequency standards, showing the effect of this dominance.

As a result, many have applied linear frequency drift corrections to their clock data, prior to performing Allan deviation calculations against their data. This technique *can* work, assuming that the resulting data will still produce convergent results. This assumption holds when the *changes* in frequency drift are

insignificant. The smaller the time span of data used, the more likely this assumption can hold true. However, several papers in previous years have confirmed that, for Block II/IIA rubidium clocks, these changes are not trivial, and hence, such an assumption often will not hold. Figure 3 plots the step-corrected frequency offsets of nine Block II/IIA rubidium frequency standards, with respect to the DoD Master Clock, over the operational lifetimes of each respective clock. The frequency drift, detectable in the general slope, appears to change somewhat randomly, and differently between the respective clocks.

Figures 4 and 5 show an NRL comparison of linear-aging-corrected Allan deviation curves and the Hadamard deviation curves for nine Block II/IIA rubidium frequency standards, using the same spans of data for each clock's respective curve. The respective spans of data exclude initial warm-up time and end-of-life times during which several of the Block II/IIA rubidium clocks experienced frequency anomalies<sup>[8]</sup>. This comparison produces three notable findings: 1) The Allan deviation plots, generally speaking, show higher values than those corresponding to the Hadamard deviation. 2) For equal  $\tau$  values, the Allan deviation often tends to *insinuate* a higher order (lower  $\alpha$ ) noise type than what the Hadamard deviation will show. 3) The presence of random walk FM, commonly thought to be high in magnitude on GPS rubidium frequency standards, does not necessarily appear as dominant in the Hadamard deviation plots for several of the frequency standards.

These discoveries suggest that the relatively high Allan deviation values for  $\tau$  values as little as 1 day and as large as 24 days appear to be significantly aliased by the non-trivial effects of changing frequency drifts. This conclusion strongly supports the use of Hadamard variance for long-term rubidium frequency standard analysis, simply because stochastic changes in frequency drift tend to show up on the Hadamard deviation as random run FM, and *not* as unwanted perturbations to stability calculations for smaller  $\tau$  values. This minimization of aliasing, therefore, permits more precise determination of the magnitude of the lower order (higher  $\alpha$ ) noise types.

## OPERATIONAL USE OF THE HADAMARD-Q EQUATION

Applying and fitting equation (2) to the data plotted in Figure 5 produces a wide variation of  $q$  values across the nine Block II/IIA rubidium frequency standards that have operated longer than four months. These variations are listed below and plotted in Figure 6.

SVN/FS#	Time Span	$q_1$ (s <sup>2</sup> /s)	$q_2$ (s <sup>2</sup> /s <sup>3</sup> )	$q_3$ (s <sup>2</sup> /s <sup>5</sup> )
16/2	Aug 89-Jan 91	3.3 E-21	3.5 E-30	6.0 E-43
19/2	Dec 94-Present	1.8 E-22	1.3 E-31	4.4 E-44
20/2	Aug 94-Jan 96	1.5 E-22	1.5 E-33	3.0 E-46
24/1	Jan 94-Jul 95	2.3 E-22	4.5 E-33	3.8 E-45
24/2	Jul 95-Present	1.0 E-22	4.2 E-32	1.1 E-43
25/2	Mar 92-Dec 93	6.0 E-22	8.0 E-33	1.0 E-45
31/2	Jan 95-Present	1.0 E-22	6.0 E-32	9.0 E-44
32/2	May 95-Aug 96	8.0 E-22	1.0 E-31	4.0 E-45
36/2	Mar 94-May 95	2.3 E-22	1.5 E-32	4.5 E-44

Note that white/flicker PM does not usually show up on either the Allan or Hadamard deviation plots for  $\tau$  values greater than or equal to 1 day. The MCS's parameter analogous to  $q_0$ , known as the measurement noise increment value, is designed to account for general modeling errors, as well as white/flicker PM. The

MCS currently uses  $0.74 \text{ m}^2$  for its measurement noise increment<sup>[4]</sup>. Operators at 2 SOPS are currently focusing on fine tuning GPS satellite ephemeris and solar state estimation. A significant refinement in this area could reduce observed periodic effects of ephemeris cross-corruption into GPS clock state modeling, and thus could permit 2 SOPS to lower their  $q_0$  values at a future date. Until such a time, personnel at 2 SOPS intend to keep this parameter fixed and equal for all satellites.

2 SOPS first used the Hadamard-Q equation operationally for the estimation of rubidium frequency standard #2 on SVN32. It's Hadamard deviation signature, based on a data span from May 1995 to May 1996, is replicated in Figure 7<sup>[9]</sup>. The  $qs$  produced from this data, as compared to the old  $qs$ , are as follows:

SVN32	Time Span	$q_1 \text{ (s}^2/\text{s)}$	$q_2 \text{ (s}^2/\text{s}^3)$	$q_3 \text{ (s}^2/\text{s}^5)$
Old $qs$ :	N/A	1.11 E-22	3.33 E-32	1.35 E-43
New $qs$ :	May 95-May 96	6.66 E-22	1.35 E-31	1.66 E-43

Note that the new  $q_1$  and  $q_2$  values were both significantly larger than the older values. Prior to the NRL's Hadamard variance analysis, 2 SOPS had no integral operational means to validate the  $qs$  that the MCS had been using for rubidium clocks<sup>[4]</sup>. The application of the Hadamard-Q equation using NRL's data has produced an unaliased, convergent result for operational use. 2 SOPS installed these new  $qs$  shortly after receipt of the report. Unfortunately, 2 SOPS had to deactivate SVN32's rubidium frequency standard #2 on 19 August 1996, shortly after the installation of these  $qs$ .

Currently, 2 SOPS makes use of  $q$  values produced by the Hadamard-Q equation for the clock state estimation of the three currently operational GPS rubidium frequency standards. SVN40's rubidium frequency standard #2, active from 1 August 1996 until 29 November 1996, did not operate long enough to produce a span of data sufficient for a Hadamard variance plot with acceptable confidence.

## CONFIDENCE INTERVALS

To exploit Hadamard variance information for fine tuning MCS  $qs$ , the Hadamard deviation plots must have large enough  $\tau$  values to show distinct signs of random run FM. As Figure 4 indicates, random run FM can start to appear for  $\tau$  values ranging between 7-24 days. One *single* calculation of the third difference, for  $\tau = 24$  days, requires 73 days of data. Furthermore, an adequate span of data is necessary to produce stability estimates with sufficiently small confidence intervals, for each respective  $\tau$  value.

The Hadamard variance requires roughly 50 % more data to produce a single stability calculation, as compared to the Allan variance, given equal  $\tau$  values. This requirement stems as a direct result of the Hadamard variance employing a third difference, as opposed to the Allan variance, which makes use of a second difference. In other words, given equal spans of data, using the Hadamard variance instead of the Allan variance reduces the degrees of freedom by one<sup>[5]</sup>. The direct implication of this freedom reduction is that the Hadamard variance requires sufficiently larger spans of data in order to produce estimates with confidence limits comparable to those produced by the Allan variance.

Generally speaking, Hadamard variance analysis requires a minimum of about 8-10 independent calculations of the third difference to produce acceptable confidence in the stability value corresponding to a given  $\tau$ . The need to calculate Hadamard deviations for  $\tau$  values of about 24 days mandates the use of

about eight months of continuous data, to produce plots usable for Block II/IIA rubidium clock  $q$  derivation.

Intuitively, this requirement imposes a challenge, given the recent operational history of rubidium clocks. As documented during PTTI '95, Block II/IIA rubidium frequency standards have typically had unexpectedly short lifetimes. To date, *not one* of the 12 activated clocks has lasted as long as *two years*<sup>[8]</sup>. By comparison, some Block II/IIA cesium clocks have outlasted the mean mission duration (six years) of the satellite. If this adverse trend continues, whenever NRL and 2 SOPS produce a set of  $q$  values suitable for use in the MCS, chances are that the rubidium clock in question may have already functioned for the *majority* of its operational lifetime. Hence, the Hadamard-Q equation can prove useful only if the frequency standard itself can operate for a sufficient duration *after* the derivation and implementation of the  $qs$  for that clock.

Note, however, that the windfall of the requirement for large spans of data is that the Hadamard variance produces confidence intervals that are *legitimate* in the presence of random run FM. Though Allan variance values may *appear* to show more confidence for GPS rubidium frequency standards, this "confidence" can prove to be misleading. The Allan variance, and for that matter, any stability metric, is valid *only* when it is convergent. Once that assumption is violated, as in the case of random run FM, the validity of the stability metric (in this case, the Allan variance) is in question.

## TIME-VARIANT DECAY OF RUBIDIUM FREQUENCY DRIFT

Since the creation of rubidium frequency standards, many analysts have employed various techniques for applying *dynamic* corrections for frequency drift, as methods for removing systematic error from clock data, prior to analyzing the clock stability using the Allan variance. William J. Riley has intelligently employed such a technique for ground-test analysis of EG&G's space-rated production rubidium frequency standard, which will serve as the primary type of frequency standard for GPS Block IIR satellites. His technique models frequency as having a logarithmic decay<sup>[10]</sup>. This type of model represents the decay of its derivative (frequency drift) as an inverse function of time, described by:

$$\text{Frequency Drift} = D_0 + AB/(B(T-T_0) + 1), \quad (3)$$

where  $D_0$  is analogous to the currently existing MCS variable for frequency drift, which can be modeled as an estimable, random walk process;  $A$  and  $B$  are coefficients tunable to the magnitude and effective rate of the decay;  $T$  is the current Kalman filter time; and  $T_0$  is the "reference time" against which the current Kalman filter time,  $T$ , is differenced, and to which the coefficients  $A$  and  $B$  are referenced.

Figure 8 shows an Allan deviation plot of several EG&G rubidium frequency standards, using data artificially corrected for the logarithmic decay modeling technique. The use of such techniques, along with the resurgence of the Hadamard variance, combined with the recently growing attention towards the Block IIR program, has initiated some very valuable discussion concerning the purpose of clock stability metrics.

The primary utility of a stability metric in GPS is its ability to quantify the *predictability* of a given frequency standard. One could think of a GPS atomic frequency standard's main purpose as its ability to physically predict time for spans ranging between 1.5 seconds and 210 days. Physical prediction is often augmented with clock modeling, as a prediction supplement. In such a scenario, as in the case of GPS, the predictability of an on-orbit clock is not only dependent on the performance of the frequency standard, but

is also limited by the sophistication of the *predictor*. Operationally speaking, the predictability of the system is only as good as the least sophisticated of the contributing elements, i.e., *a chain is only as strong as its weakest link*.

The current GPS estimation architecture relies on a three-state polynomial clock model, whose prediction during time updates uses the following polynomial expansion<sup>[1]</sup>:

$$\begin{bmatrix} x(t+\tau) \\ y(t+\tau) \\ z(t+\tau) \end{bmatrix} = \begin{bmatrix} 1 & \tau & (1/2)\tau^2 \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (4)$$

where  $\tau$  is the prediction span, and  $x(t)$ ,  $y(t)$ , and  $z(t)$  are the phase, frequency, and frequency drift values, respectively, of the clock in question. Note that  $y(t)$  is the time derivative of  $x(t)$ , and  $z(t)$  is the time derivative of  $y(t)$ .  $\Delta(x)$ ,  $\Delta(y)$ , and  $\Delta(z)$  are random error increments, independent of  $x(t)$ ,  $y(t)$ , and  $z(t)$ , having a prediction covariance  $P$  represented by a function of the Kalman filter process noises ( $q_s$ )<sup>[4]</sup>:

$$P = E \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \begin{bmatrix} \Delta x & \Delta y & \Delta z \end{bmatrix} = \begin{bmatrix} q_1\tau + q_2\tau^3/3 + q_3\tau^5/20 & q_2\tau^2/2 + q_3\tau^4/8 & q_3\tau^3/6 \\ q_2\tau^2/2 + q_3\tau^4/8 & q_2\tau + q_3\tau^3/3 & q_3\tau^2/2 \\ q_3\tau^3/6 & q_3\tau^2/2 & q_3\tau \end{bmatrix} \quad (5)$$

Note that, currently, *no* parameters representing any type of time-variant decay of frequency drift exist within the GPS estimation architecture. EG&G has successfully demonstrated how a logarithmic model can well represent the major systematic component of a changing frequency drift<sup>[11]</sup>. Additionally, another available decay modeling technique is the exponential model. Implementation of an exponential model would involve the inclusion of additional Kalman filter parameters, used as follows<sup>[12]</sup>:

$$\text{Frequency Drift} = D_0 + D_1[e^{**}\{D_2(T-T_0)\}], \quad (6)$$

where  $D_0$  is the currently existing MCS variable for frequency drift, which effectively models frequency drift as an estimable, random walk process;  $D_1$  and  $D_2$  are coefficients designed to represent the initial magnitude and effective rate of decay, respectively;  $T$  is the current Kalman filter time; and  $T_0$  is the "initialization time" against which the current Kalman filter time,  $T$ , is differenced.

Nonetheless, until such a time that GPS can take advantage of logarithmic or exponential compensation for systematic changes in frequency drift, the Hadamard variance is the *sole* available metric that best represents the predictability of rubidium frequency standards in GPS. A recent Hadamard variance analysis of an EG&G space-rated rubidium frequency standard, shown in Figure 9, suggests that, even using MCS prediction, Block IIR rubidium frequency standards look *very* promising in terms of stability/predictability<sup>[11]</sup>.

## CONCLUSIONS

1. The GPS program would benefit from retaining the availability of the three random walk degrees of freedom for satellite Kalman filter clock estimation.
2. The GPS program should earnestly consider the incorporation of time-variant decay parameters, as described earlier, as *additional* degrees of freedom in GPS clock estimation architecture.
3. GPS clock stability plots using data preliminarily corrected for systematic errors, such as the logarithmically corrected Allan deviation, are useful for identifying the possible benefit of future systematic models within GPS estimation architecture.
4. The use of a clock stability metric for identifying the predictability of a clock within a given system is most representative when the metric corresponds to the predictor/estimator involved. As such, the Hadamard variance best corresponds to current GPS rubidium clock estimation architecture.
5. Analysts who present stability plots at various symposia can greatly help their audience by identifying their purpose behind their respective choices of stability algorithms. Though a systematically compensated stability plot may be useful for identifying removable systematics, it may prove misleading when trying to present performance metrics with a reasonable degree of fairness.
6. Generally speaking, the need to obtain sufficient confidence in the magnitude of random run FM, present in GPS rubidium clocks, necessitates using at least eight months worth of data, after initial clock warm-up.
7. The importance of the Hadamard variance will continue to increase as more GPS satellites operate rubidium frequency standards. The operational start of Block IIR program will induce a sharp increase in the use of rubidium frequency standards. This increased usage will further the importance of optimized extended autonomous navigation and timing predictions. Inaccurate estimation of frequency drift could result in hundreds of meters of ranging error, and microseconds of time transfer error during autonomous operations<sup>[13,14]</sup>. With the first Block IIR launch scheduled for 1997, the role of the Hadamard variance will increase dramatically.
8. If GPS chooses to implement time-variant frequency drift decay models at a future date, any residual existence of unmodeled random run FM will emphasize the need for continued use of  $q_3$ , and, thus, the Hadamard variance.

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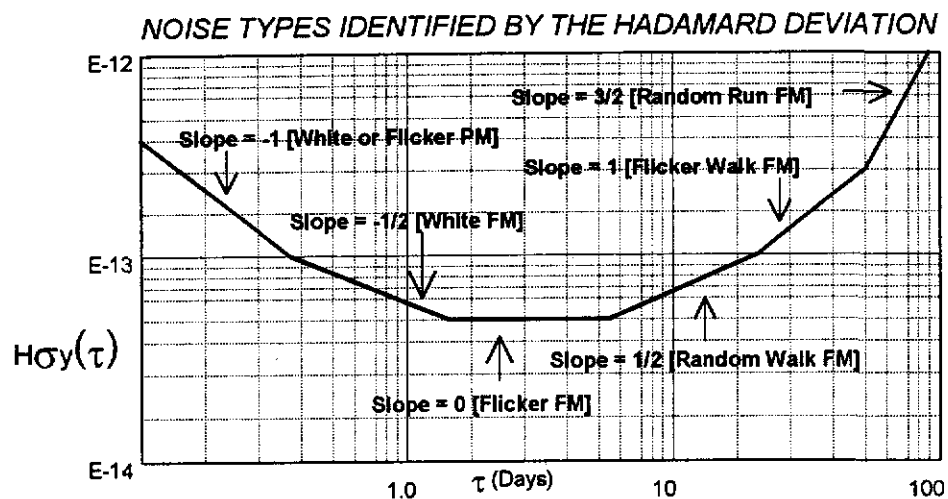
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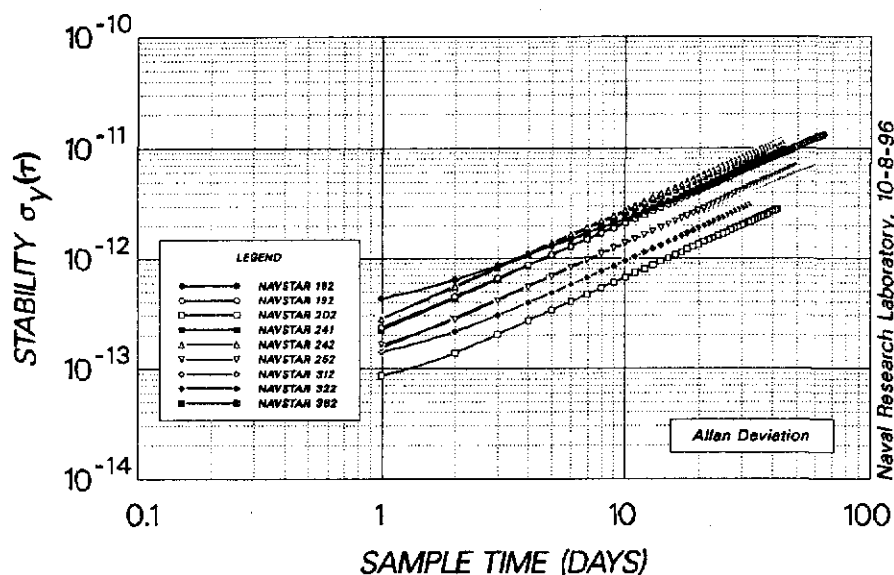
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Noise Type	Power-Law Spectral Density Exponent	$H\sigma_y(\tau)$ Log-Log Slope	Process Noise Parameter
White PM	$\alpha = 2$	-1	$q_0$
Flicker PM	$\alpha = 1$	-1	$q_0$
White FM	$\alpha = 0$	-1/2	$q_1$
Flicker FM	$\alpha = -1$	0	
Random Walk FM	$\alpha = -2$	1/2	$q_2$
Flicker Walk FM	$\alpha = -3$	1	
Random Run FM	$\alpha = -4$	3/2	$q_3$

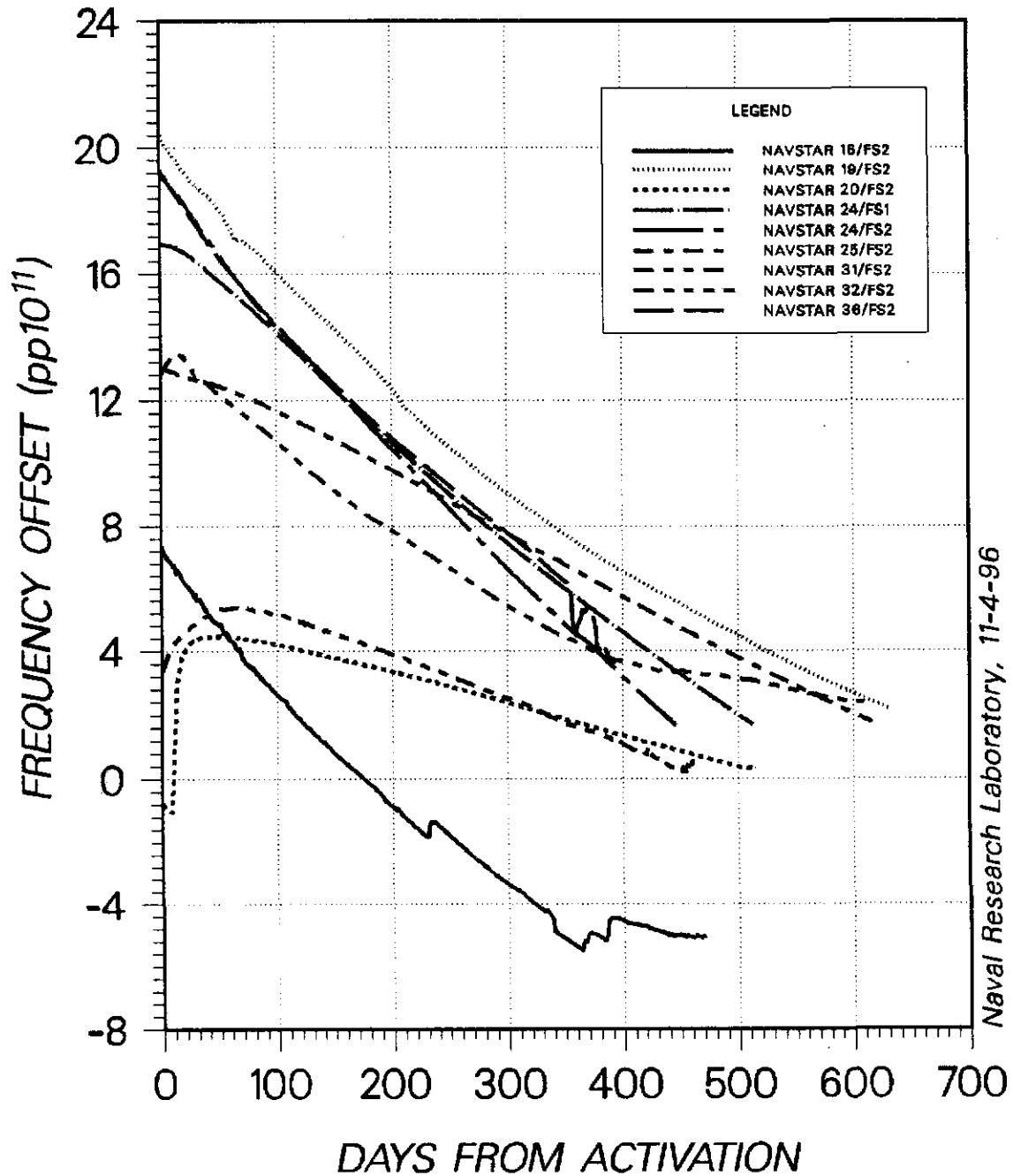
**FIGURE 1**

**FREQUENCY STABILITY OF BLOCK II/IIA RUBIDIUM CLOCKS  
REFERENCE DOD MASTER CLOCK  
AGING UNCORRECTED**



**FIGURE 2**

**BLOCK II/IIA RUBIDIUM CLOCKS  
CORRECTED FREQUENCY OFFSET FROM  
DOD MASTER CLOCK**



**FIGURE 3**

FREQUENCY STABILITY OF BLOCK II/IIA RUBIDIUM CLOCKS  
REFERENCE DOD MASTER CLOCK  
AGING CORRECTED

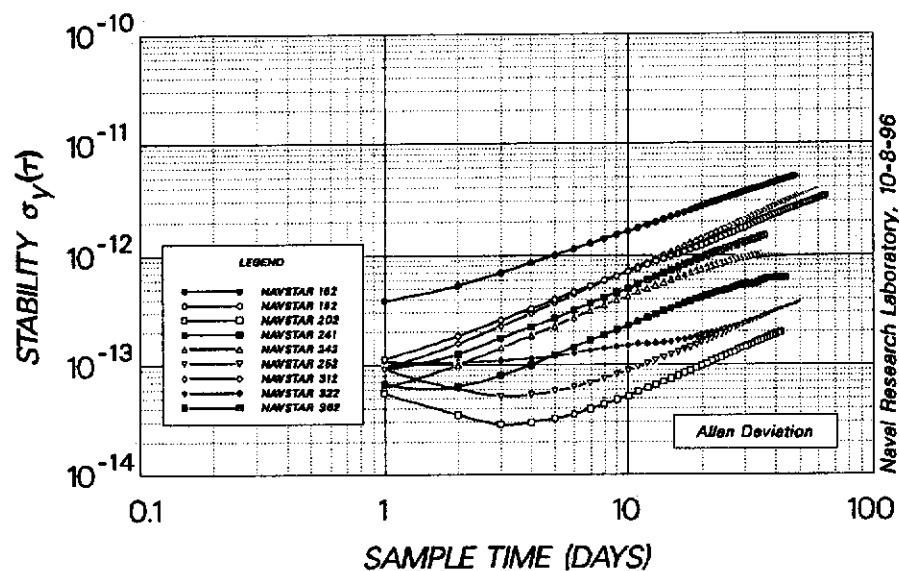


FIGURE 4

FREQUENCY STABILITY OF BLOCK II/IIA RUBIDIUM CLOCKS  
REFERENCE DOD MASTER CLOCK

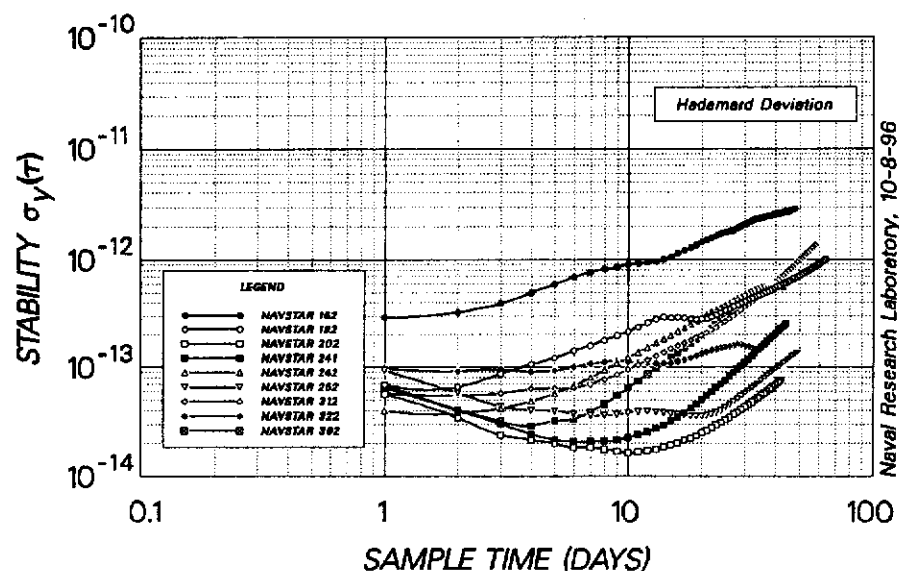
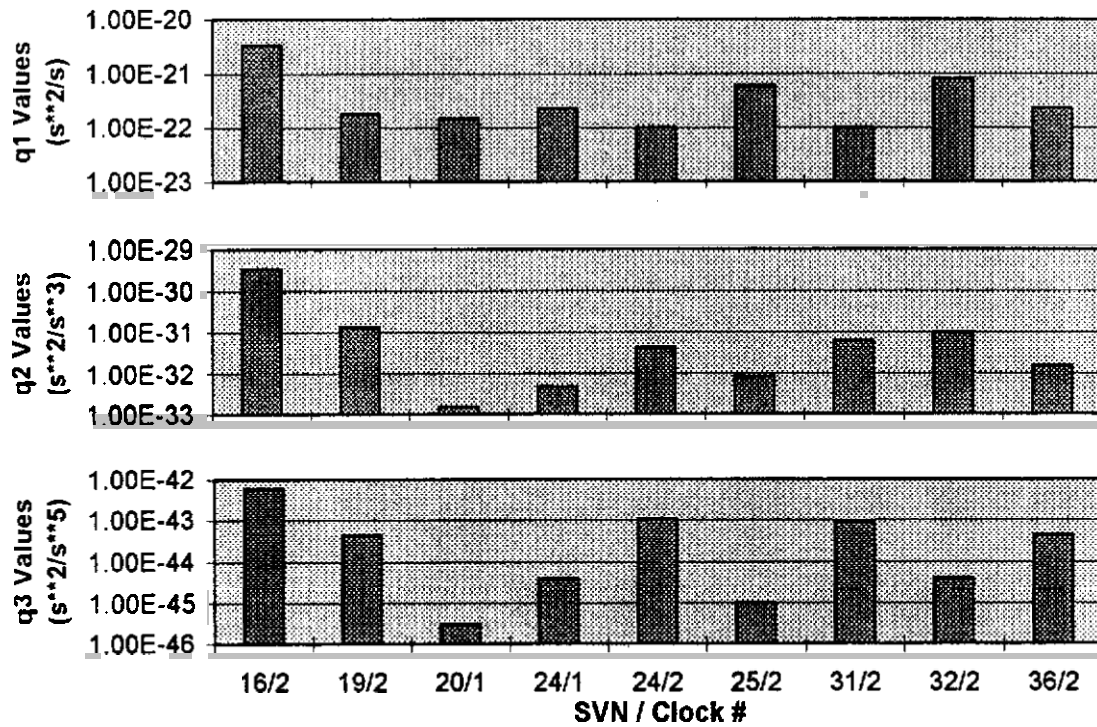


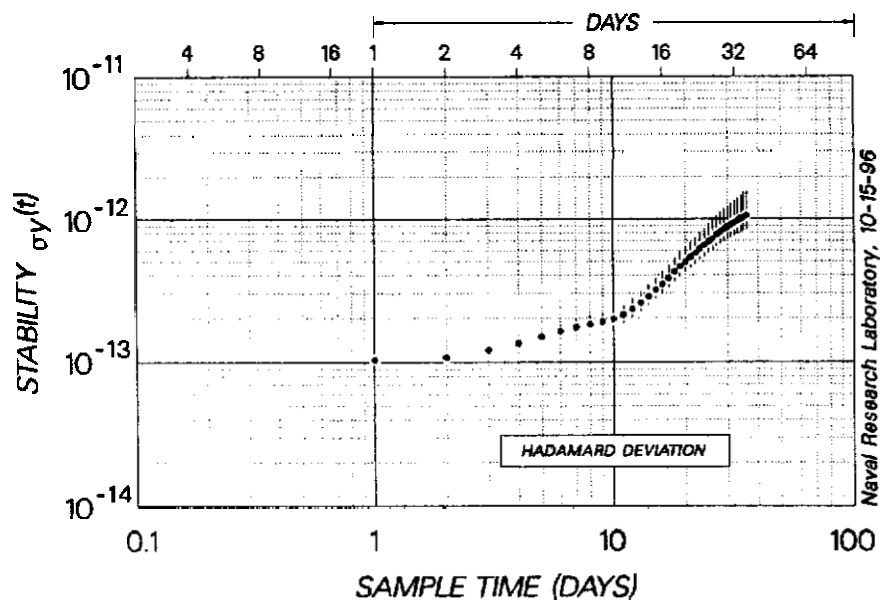
FIGURE 5

### PROCESS NOISE COMPARISON



**FIGURE 6**

FREQUENCY STABILITY OF RUBIDIUM OSCILLATOR (NO.85)  
 NAVSTAR 32 (Block II-16, Plane F-1)  
 DoD Master Clock (PPS)  
 13-MAY-95 to 4-MAY-96



**FIGURE 7**

# ALLAN DEVIATION: LOGARITHMIC FREQUENCY DECAY REMOVED

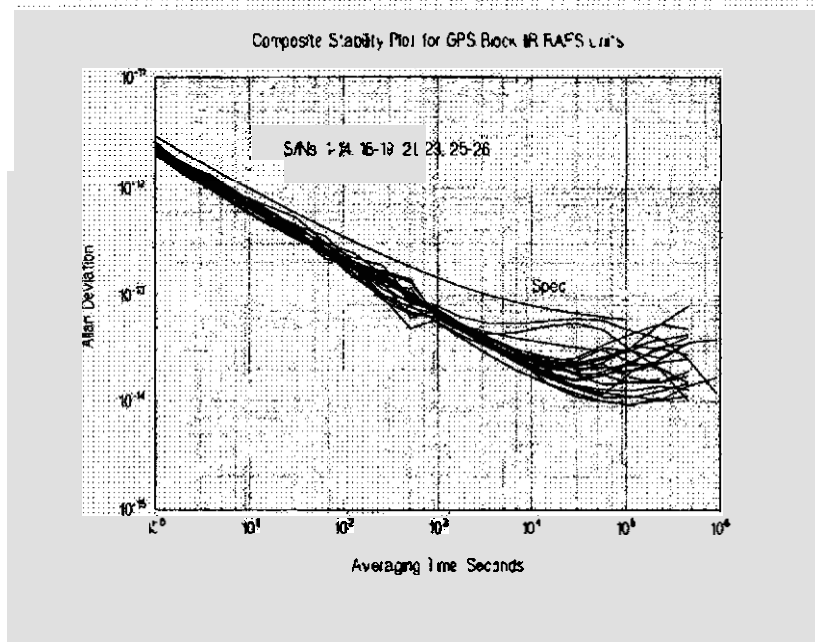


FIGURE 8

FREQUENCY STABILITY OF RUBIDIUM OSCILLATOR (NO.4)  
EG&G, INC.  
HYDROGEN MASER (NO.N1)  
25-OCT-95 to 11-APR-96

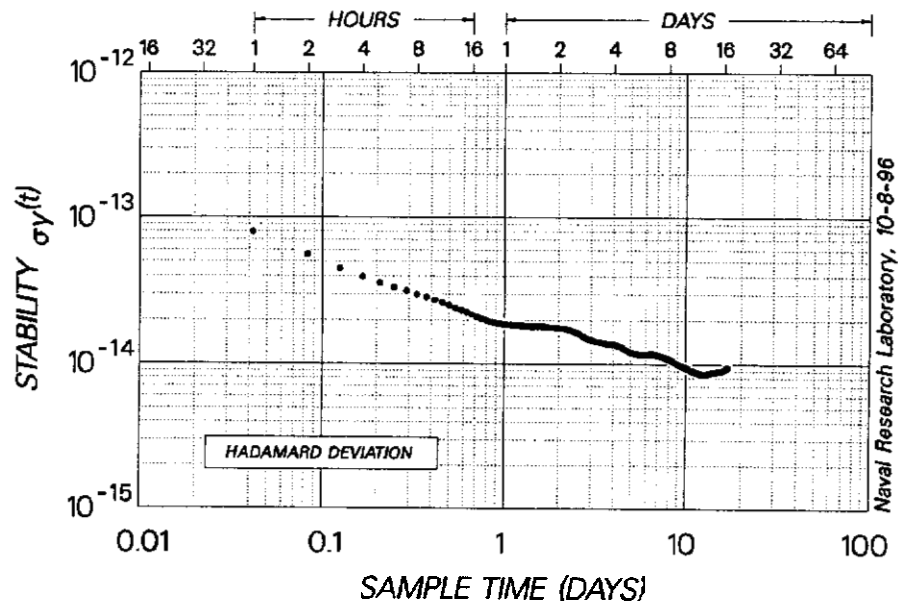


FIGURE 9

## Questions and Answers

DAVID ALLAN (ALLAN'S TIME): I would like to commend you, Steve, for an outstanding paper and work. I think this is very commendable. I think with the change of the ability to characterize each individual clock in the constellation, we should see a significant improvement in the composite clock stability. And this should be very noticeable. We should see the long-term noise performance of the composite clock go down.

One question: Have you seen a logarithmic behavior on the orbit rubidiums? Have you tried to model those?

STEVEN HUTSELL: Let's put this plot up. It's hard to tell; I think in a couple of them, yes, you could probably very easily fit a logarithmic or exponential curve that will probably take out most of the rate of change. But on the others – you look at SVN 31, I believe it is – I don't know if that's something that would work best.

And the other challenge with that is we have to know what the coefficients of a logarithmic equation are ahead of time because we're doing real time or future prediction. It's a challenge. My general opinion about that is, I think the majority of systematic change on the IIR rubidiums is like what you said, a decay. But for II and IIAs, I think there's more noise occurring in it than systematic.

DAVID ALLAN: The other important point, I think, as we go into autonomous mode and long-term, if you even want to think about 180-day or 60-day autonomy, then your ability to predict behavior over much longer time is very important, having these long-term characterizations nailed down. So that should impact the autonomy as well.

The last point I'd like to make is the one you really made, but I'd like to hammer home with a big nail. That is that we ought not to model systematics with statistics. If we can get hold of a systematic, we should use our best estimator to deal with it because we'll do much better in prediction if we can model it using logarithmic or whatever best kind of modeling deals with the systematic and not do it statistically.

CARROLL ALLEY (UNIVERSITY OF MARYLAND): The short-term stability of rubidium standards, from the point of view of the user, needs to be emphasized, so that if you can go more to the rubidium standards with the modeling that you've suggested, it could have this other advantage as well.

STEVEN HUTSELL: Yes, our hope is that the reliability of the IIR rubidiums will surpass that of the II and IIA. There was a great paper presented last year by Capt. Greg Hatten and Lt. Gary Dieter discussing what was previously not really well known to the community; and that was the somewhat disappointing reliability of rubidium frequency standards in GPS.

But you're right. If you look at this – as a fair comparison, if you have white FM, given enough data to produce enough confidence, the Hadamard variance and the Allan variance will converge to the same number. That's only for white FM.

But as a comparison, you're seeing some frequency standards with stability on the order of 4 parts in 10 to the 14th, which is exceptional. And as you saw, the EG&G rubidium on the order of 2 parts in 10 to the 14th. Yes, I would agree.